Factor modeling

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Outline

1. Factors of production
2. Capital modeling
3. Land modeling
4. Labor modeling

Factors of production

Factors of production

- ***Factors of production*** – goods that need to be purchased by the company to ensure the release of other goods – finished products.
- The main types of factors:
  - capital,
  - labor,
  - land (natural resources),
  - entrepreneurial skills.
- The prices of factors of production are, respectively, ***interest, wage rate, rent*** and ***normal income.***

Basics – 1

- **The marginal cost of the resource** (*MRC*) – is the additional cost of production with increasing resource use per unit:

$$MRC = \Delta TC / \Delta X$$

- **Marginal product in monetary terms** (*MRP*) – is the additional income from the sale of additional products produced through the use of an additional unit of resource:

$$MRP = MP \cdot MR$$

where MP– marginal product,
MR – marginal revenue.

Basics – 2

- The optimal use of the factor required to maximize the company's profits must satisfy the condition:

$$MRC = MRP$$

- Production costs will be minimal if:

$$\frac{MRP_1}{p_1} = \frac{MRP_2}{p_2} = \dots \frac{MRP_k}{p_k}$$

where p_i is the factor price.

Capital modeling

Capital market

- **Capital** – is a productive resource of long use; any goods created in contrast to labor and land, which are used in the production of other goods.
- **Capital markets** – These are markets in which capital is physically borrowed on various terms.

Features of capital goods

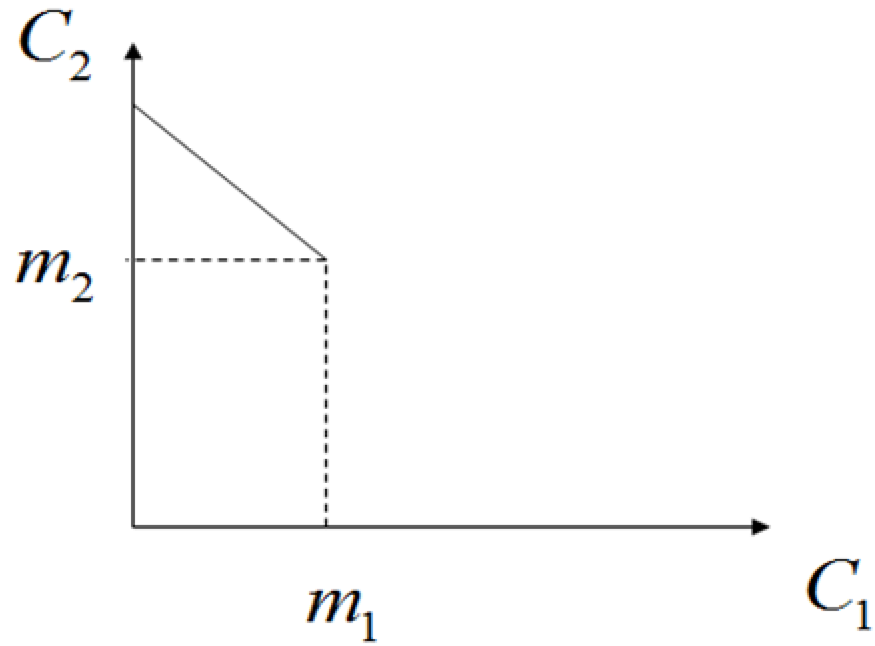
- Long period of use, so the decisions of firms to attract them should always take into account the time factor;
- it is both a factor of production and a product at the same time.

Model assumptions

- The individual decides how much goods to consume in each of the two time periods;
- commodity prices are constant and equal to 1;
- (c_1, c_2) – the amount of consumption in each period;
- (m_1, m_2) – the amount of money of the individual in each period.

Model – 1

- If the individual if it is not possible to borrow money, then the maximum amount that he can spend in period 1 is m_1.



Model – 2

- The consumer can borrow and lend at a rate of interest r.
- Prices are constant at level 1.

- Consumer–creditor:

$$c_2 = m_2 + (m_1 - c_1) + r(m_1 - c_1)$$
$$= m_2 + (1 + r)(m_1 - c_1)$$

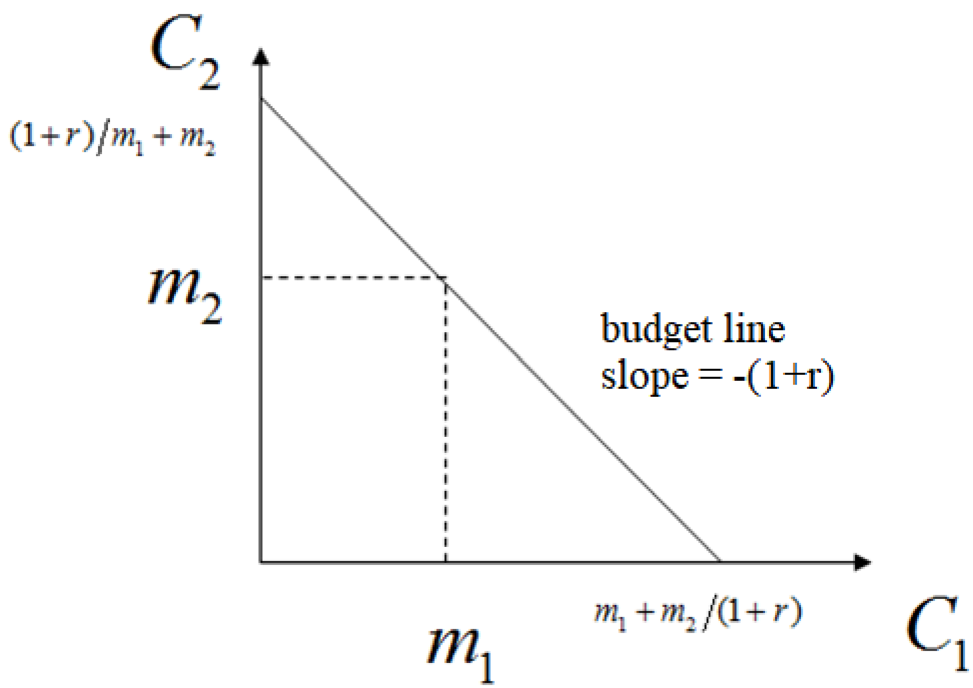
- Consumer–borrower $(c_1 > m_1)$:

$$c_2 = m_2 - r(c_1 - m_1) - (c_1 - m_1)$$
$$= m_2 + (1 + r)(m_1 - c_1)$$

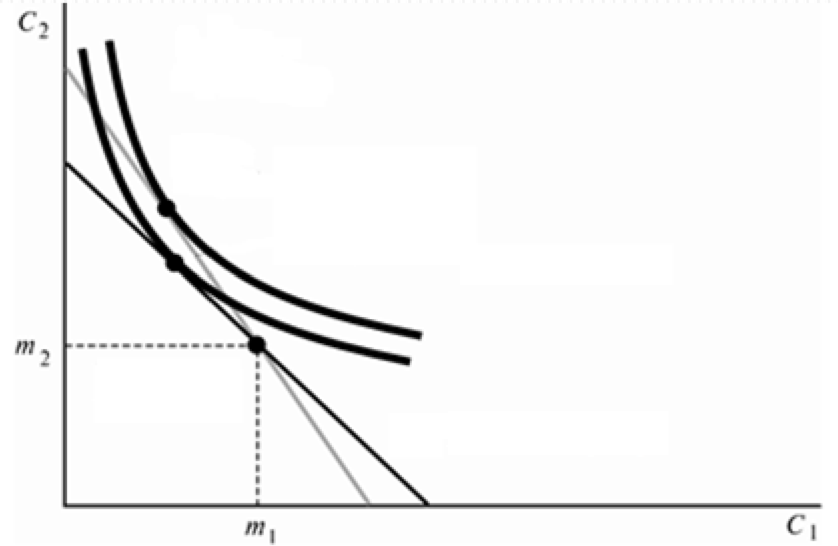
Budget constraint

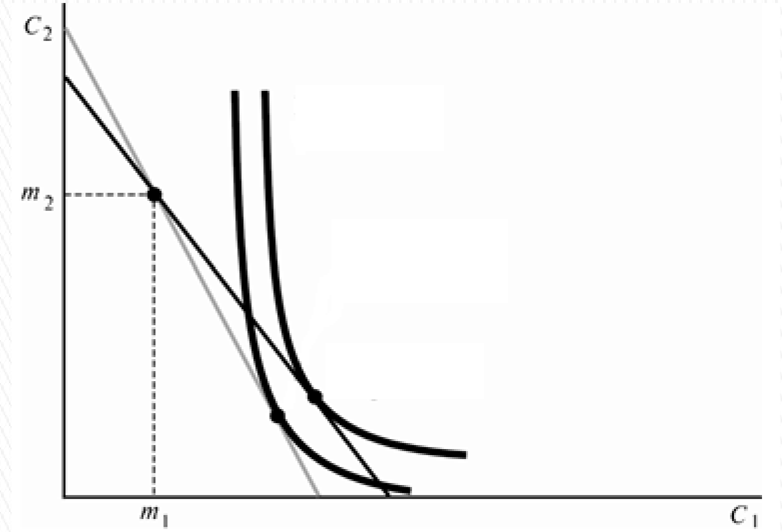
$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

$$c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}$$



Rate change





If this individual is a lender and the interest rate rises, he will remain a lender.

When the interest rate for the borrower rises and he decides to remain a borrower, his welfare decreases.

Slutsky equation– 1

- Assume that the interest rate rises.

$$\frac{\Delta c_1^t}{\Delta p_1} = \frac{\Delta c_1^s}{\Delta p_1} + (m_1 - c_1)\frac{\Delta c_1^m}{\Delta m}$$

(?) (–) (?) (+)

- The usual equation Slutsky looks like:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} + (w_1 - x_1)\frac{\Delta x_1^m}{\Delta m}\Delta,$$

where $\frac{\Delta x_1^s}{\Delta p_1}$ – substitution effect,

$\frac{\Delta x_1^m}{\Delta m}$– income effect.

Equation analysis

- For the borrower, raising the interest rate means that he will have to pay a higher interest rate tomorrow. This encourages him to borrow less and thus consume less in the first period.
- For the lender, the considered effect is ambiguous. From the lender's point of view, rising interest rates can bring him so much extra income that he will even want to increase his consumption in the first period.

Slutsky equation– 2

$$\frac{\Delta x_1^S}{\Delta p_{\mathbf{1}}} = \frac{x_{\mathbf{1}}(p'_1, m') - x_{\mathbf{1}}(p_1, m)}{\Delta p_{\mathbf{1}}}$$

$$\frac{\Delta x_1^m}{\Delta m} x_1 = \frac{x_1(p'_1, m') - x_1(p', m)}{m' - m} x_1$$

$x_1(p_1, m)$– consumer consumption function;

p – the price of the goods;

m – consumer income;

$\Delta = m' - m = x_1(p_1' - p_1)$– change in income

Slutsky equation – 3

- Let the price changes to the interest rate (r + 1), then:

$$\frac{C_1(1+r',m)-C_1(1+\mathrm{r},m)}{r'-r}=$$

$$=\frac{C_1(1+r',m+\Delta m)-C_1(1+r,m)}{r'-r}+$$

$$+(m-C_1)*\frac{C_1(1+r',m+\Delta m)-C_1(1+r',m)}{\Delta m}\Bigg)$$

where $C_1(1+r,m)$– consumer consumption function.

Substitution and income effects

- As the interest rate rises, the opportunity cost of current consumption increases.
- In the change in the ratio between consumption and savings due to the growth of the opportunity cost of current consumption with a constant level of consumer welfare has a manifestation of the substitution effect.

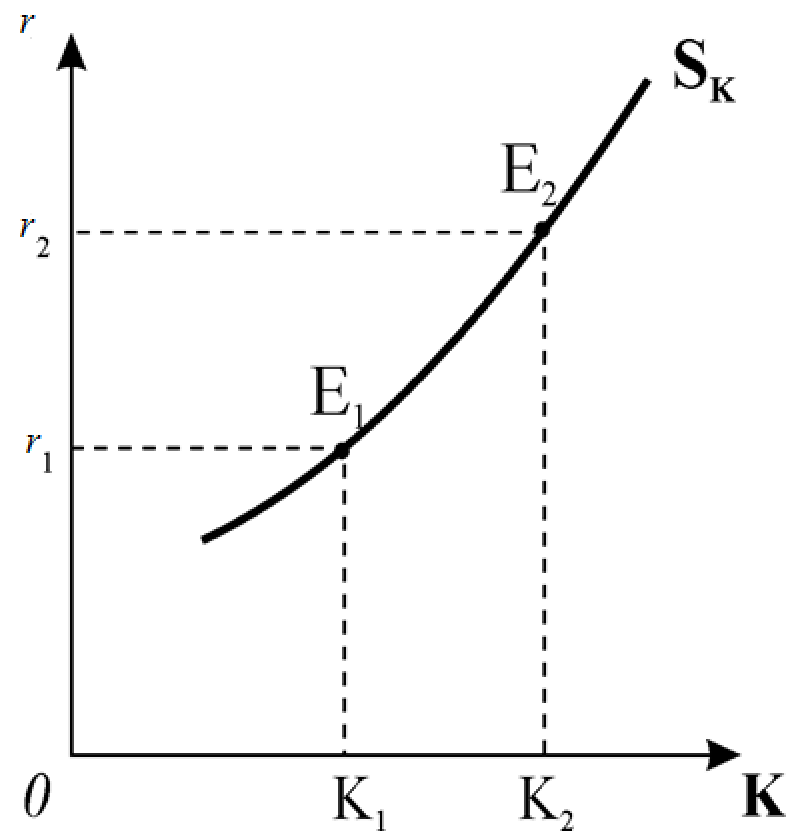
- An increase in the interest rate increases the volume of future consumption, which reduces its relative value to man and stimulates to reduce the level of savings.
- In the change in the ratio between current and future consumption in response to the growth of total (current and future) income due to rising interest rates has a manifestation of the effect of income.

Substitution effect

Income effect

Loan market supply curve

- Is formed as the horizontal sum of the curves of individual loan supply.
- Empirical studies show a low elasticity between the interest rate and the amount of savings, which means an almost vertical shape of this curve.



Example

- The individual lives in two periods. Its utility is given by the formula:

$$U_t = \frac{C_1^{1-\theta}}{1-\theta} + \frac{1}{1+\rho}\frac{C_2^{1-\theta}}{1-\theta}$$

where

- $\theta > 0$ coefficient of interperiod advantages,
- $\rho > -1$(discount rate).

- Let $P1 = 3$ and $P2 = 4$ – consumption prices in two periods, W = 30 – income individual for life.
- What will be equal C_1 and C_2?

Solution

$$3C_1 + 4C_2 = 30.$$

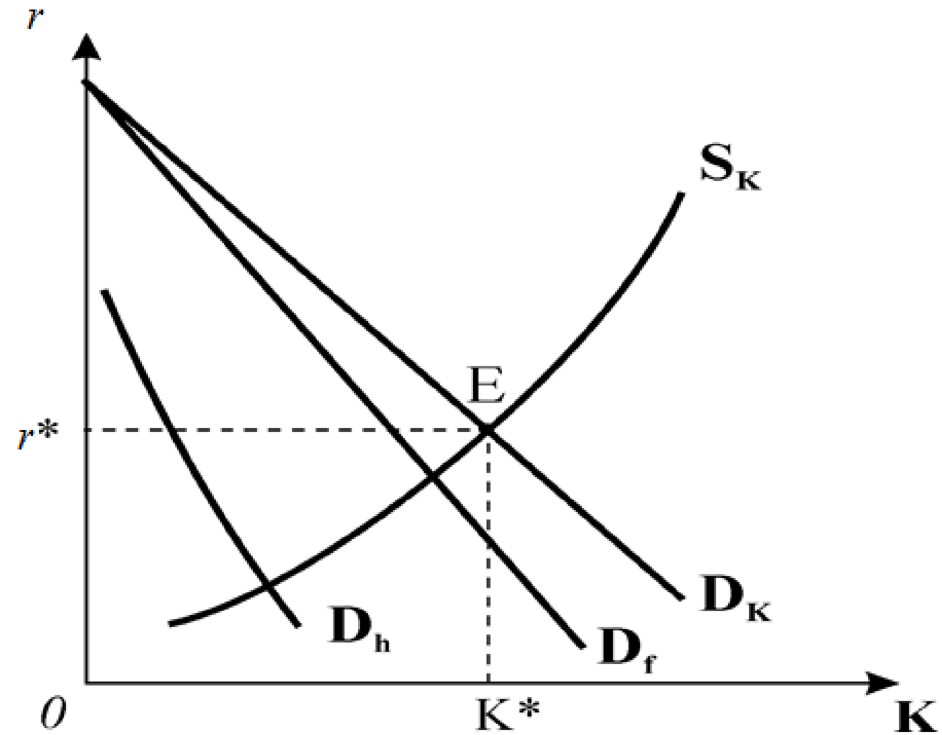
$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2} \Rightarrow$$

$$\frac{C_1^{-\theta}}{3} = \frac{C_2^{-\theta}}{(1+\rho)4} \Rightarrow C_1 = \left(\tfrac{4}{3}(1+\rho)\right)^{-\theta} C_2$$

$$C_2 = \frac{30}{3\left(\tfrac{4}{3}(1+\rho)\right)^{-\theta} + 4} \qquad C_1 = \frac{30}{3 + 4\left(\tfrac{4}{3}(1+\rho)\right)^{-\theta}}$$

Demand in the capital market

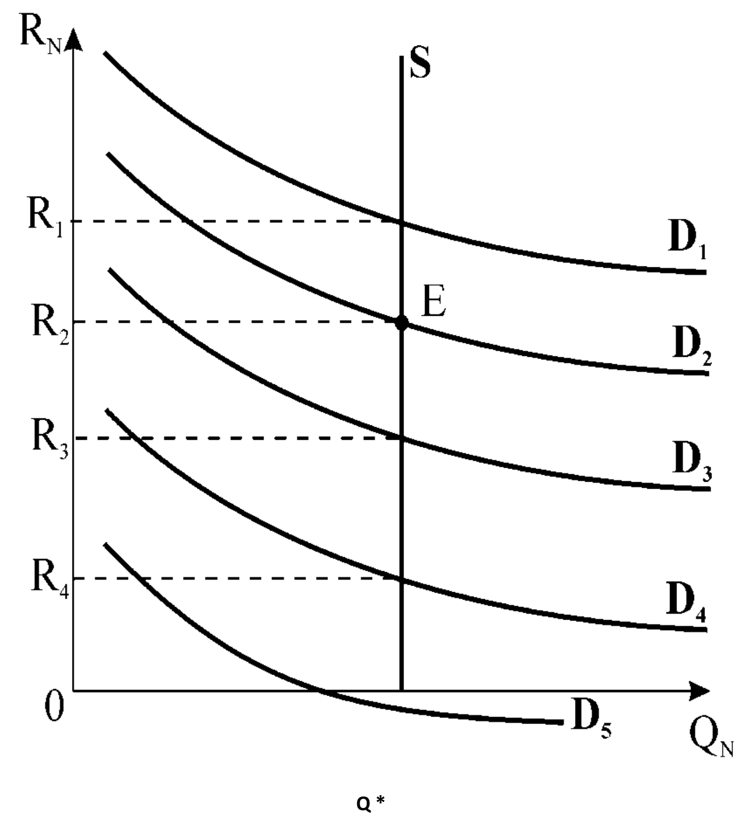
- Demand for borrowed funds has two components:
  - (D_f) – demand of firms;
  - (D_h) – household demand.



Land modeling

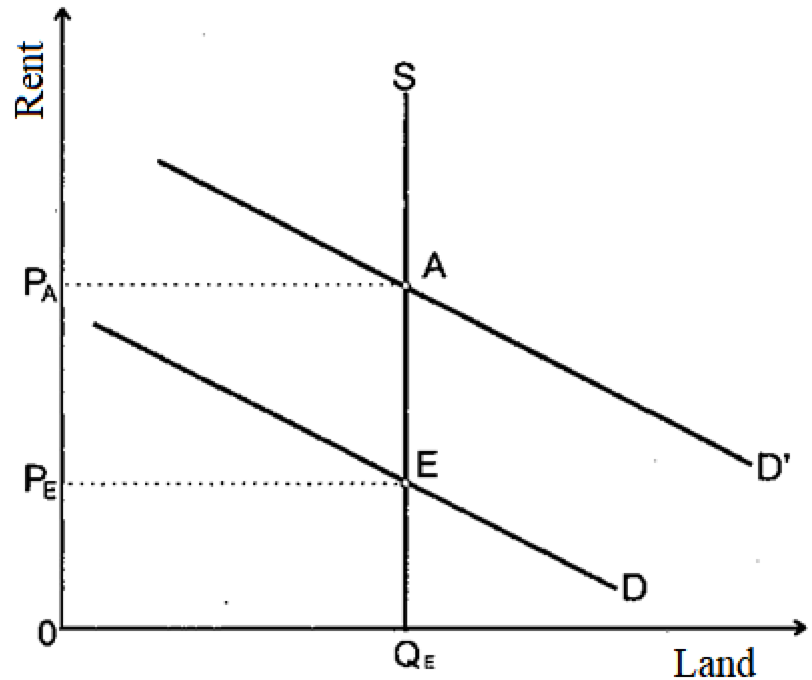
Land and natural resources market

- The supply of land is completely inelastic, so the price of land (land rent) depends only on changes in demand for it.



Land market with two types

- D – agricultural demand;
- D'– non-agricultural demand – economic (demand for land for housing, industrial use, etc.).



Land and natural resources market

- The price of land as a perpetual asset is a capitalized land rent

$$P_N = (R_N / r) \cdot 100\%$$

- The land is sold for such an amount that in the case of its alternative use will bring income equal to the land rent.

Labor modeling

Labor

- ***Labor*** (or *labor services)* – one of the main factors of production owned by households; it is the physical and mental abilities of people that can be used in the production of goods.
- The uniqueness of labor as a factor of production is that labor services cannot be separated from the worker.

Market labor

- *Labor market* – the market of one of the factors of production, where households in the role of employees offer their work, and firms-producers of goods and services (employers) – need it. The labor market sets the price of labor – the wage rate – and the volume of labor use.

Labor market assumption

- ***absolute labor mobility*** – the ability of workers to move their labor services from one firm to another indefinitely, regardless of the specialization of labor markets.

Income-leisure model

- *L* – daily workload in hours;
- *(24-L)* – rest for a day;
- *I* – income per day;
- income and rest are normal benefits.

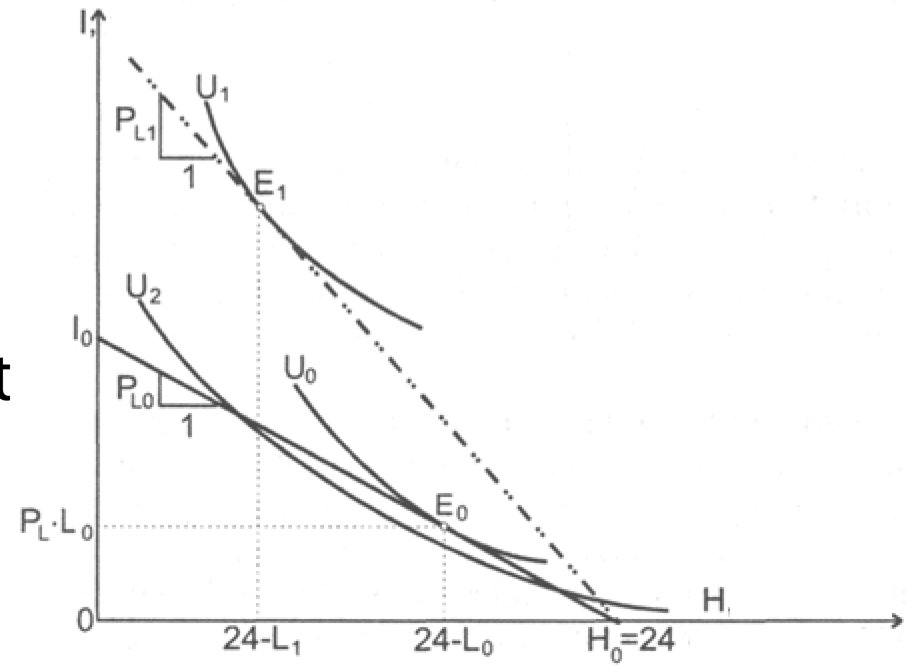
$$U = f\left(I, 24 - L\right)$$

$$I = P_L * L$$

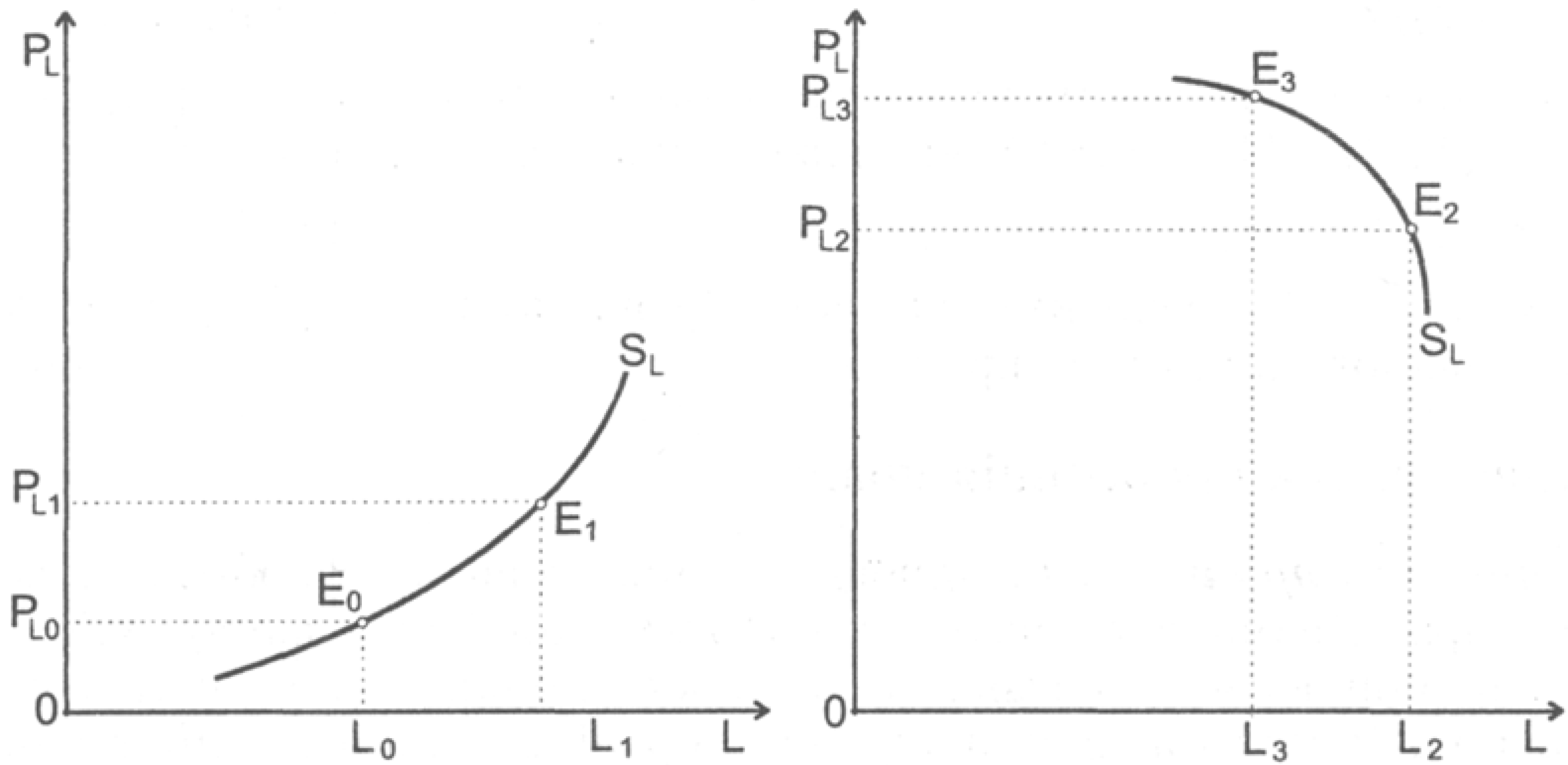
$$0 \leq L \leq 24$$

Income and substitution effects

- The effect of income arises from increasing welfare (real income) with increasing wage rates (this encourages less work).
- The substitution effect arises from the increase in the opportunity cost of an hour of rest (which stimulates an increase in working hours).

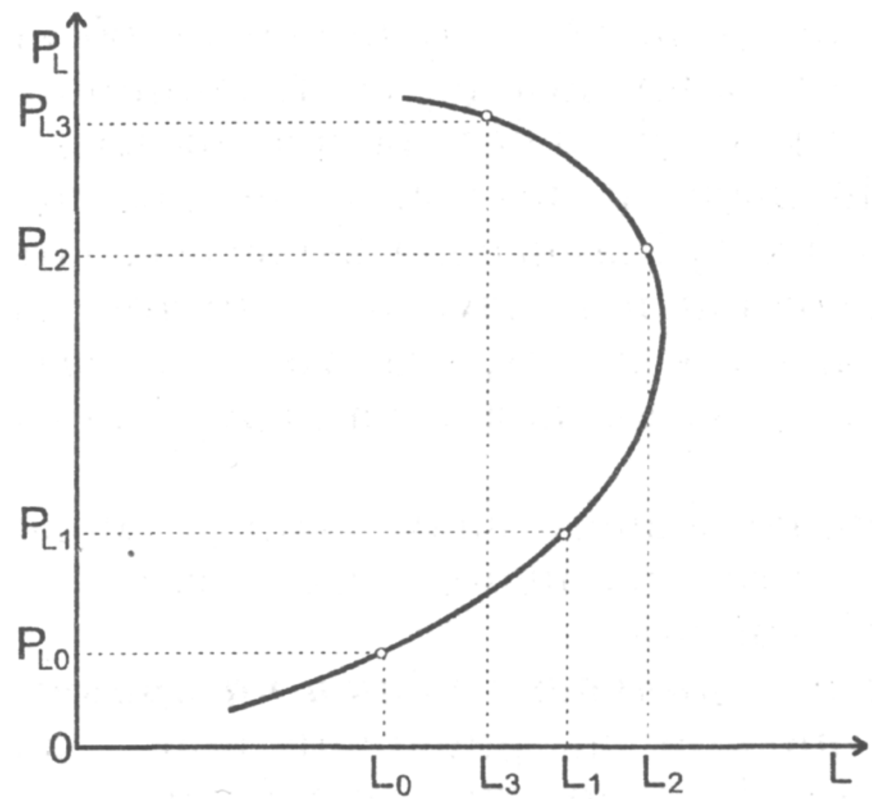


Labor supply curve



Empirical research

- Empirical studies show that at low wage rates the substitution effect prevails, at higher rates the income effect prevails.



Individual welfare model: model assumptions

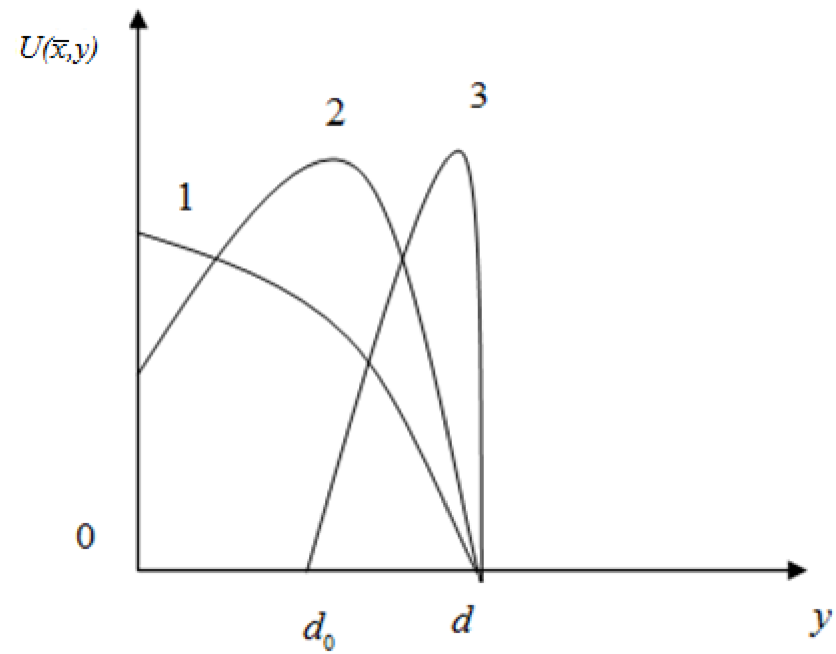
- The main source of income is labor;
- the individual works to maximize well-being.

Basics of the model

- y – labor intensity,
- q – hourly wage rate
- x – the number of goods consumed,
- p – prices for goods,
- a – minimum cost of living,
- r – rent from the property,
- γ – tax rate,
- d – limit of physical capabilities,
- α – propensity to consume,
- U – well-being function of the individual

Propensity to work

- 1– work is an unpleasant burden,
- 2 – work is a necessity.
- 3 – work is a necessity.



Model description – 1

$$\begin{cases} \varphi(x,y) = (x-a)^{\alpha}(d-y)^{1-\alpha} \to \max \\ a \le x \le (1-\gamma)qy + r \\ 0 \le y \le d \\ 0 < \alpha < 1 \\ 0 \le \gamma \le 1 \end{cases}$$

- Because $\varphi(x,y)$ grows on x, then

$$x^{*} = (1-\gamma)qy^{*} + r$$

Model description – 2

$$a \le (1-\gamma)qy + r \Rightarrow \gamma \le 1 - \frac{a-r}{qd}$$

$$\begin{cases} \varphi(y) = ((1-\gamma)qy + r - a)^{\alpha}(d-y)^{1-\alpha} \to \max \\ \frac{a-r}{q} \le y \le d \\ \gamma \in \left[0; 1 - \frac{a}{qd}\right] \\ 0 < \alpha < 1 \end{cases}$$

$$\varphi'(y) = 0 \Rightarrow y^{*} = \alpha d - \frac{(1-\alpha)(r-a)}{(1-\gamma)q} \Rightarrow$$

$$x^{*} = (1-\gamma)\alpha qd - (1-\alpha)(r-a) + r$$

Model analysis

- when α is growing φ^* grows, i.e. the desire to work of citizens enriches the state;
- growing of d, which determines the health of nation, increases φ^*, and therefore the state of sick people can not be rich;
- when a is growing φ^* is growing, so a high subsistence level is typical for highly developed countries;
- growing q leads to increase of φ^*, so in highly developed countries always expensive labor and vice versa;
- if $r < a$, the formula of the model loses its meaning, so high incomes to the state can provide only wealthy citizens.

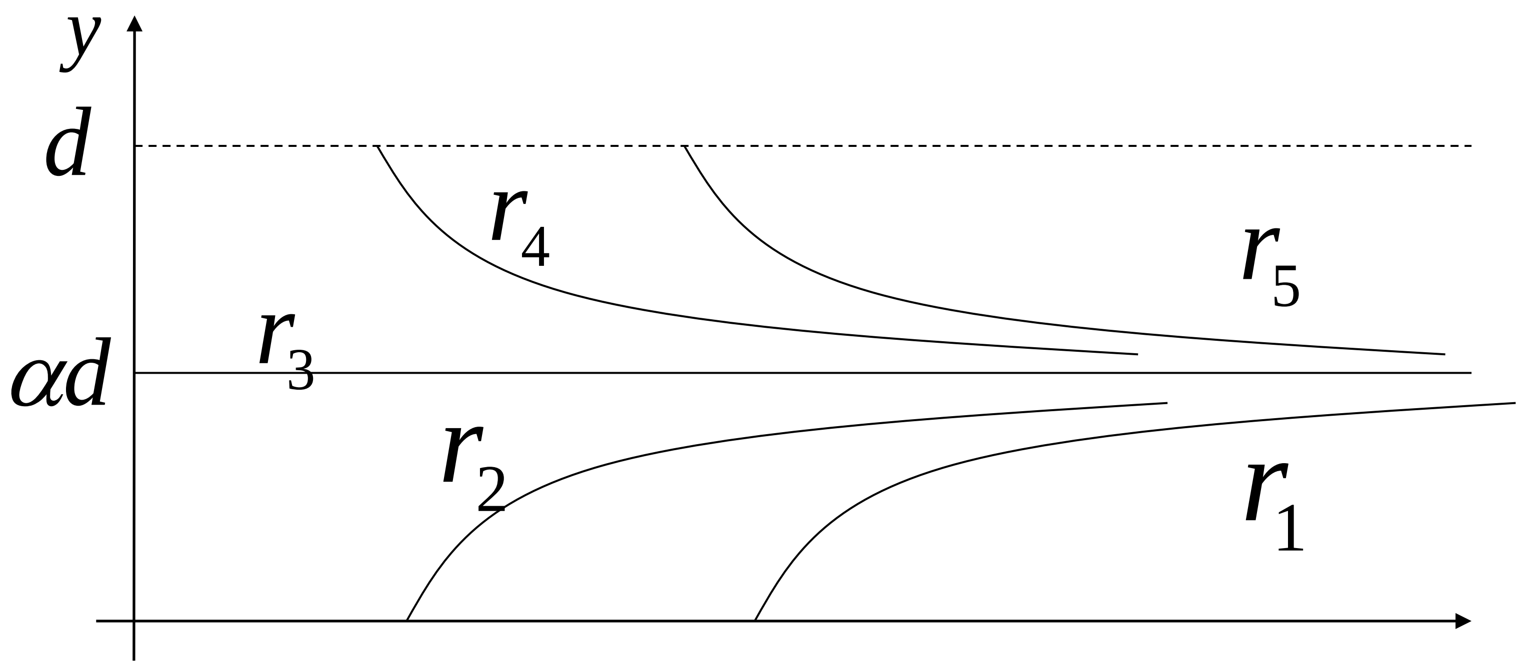
Analysis of labor activity depending on consumer rent – 1

$$\varphi(x,y) = (x-a)^{\alpha}(d-y)^{1-\alpha}$$

$$\begin{cases} y^* = ad + \dfrac{(1-\alpha)(a-r)}{q} \\ x^* = qy^* + r \end{cases}$$

$$\varphi^*(q,r) = (1-\alpha)^{\alpha}\left[d + \frac{r-a}{q}\right]$$

Analysis of labor activity depending on consumer rent – 2



$$r_1 > r_2 > r_3 = a > r_4 > r_5 = 0$$

Conclusions of the analysis

- If the income from property is less than the subsistence level, then wages act as a destimulating factor
- If the rent is equal to the subsistence level, the labor activity does not depend on the level of wages
- If the rent is more than the subsistence level, then wages stimulate labor activity

Comparison of individual and collective work

- N – number of employees in the team
- y_s – labor intensity $s$employee
- q_s – payment per unit of labor intensity for employee

Model of labor activity s^th employee – 1

- If the individual works for himself, then

$$\begin{cases} Y_s(x_s, y_s) \to \max \\ a \le x_s \le q_s y_s \\ 0 \le y_s \le d \\ s = \overline{1, N} \end{cases}$$

Model of labor activity s^th employee – 2

- If there is an equal distribution between team members, then

$$\begin{cases} \varphi_s(x_s, y_s) = (x_s - a)^{\alpha} (d - y_s)^{1-\alpha} \to \max \\ a \le x_s \le \dfrac{1}{N} \sum_{i=1}^{N} q_i y_i \\ 0 \le y_s \le d \end{cases}$$

Optimal solution

- Since the welfare function is increasing on x_s, x_s^* it will take the value of the upper limit of the interval:

$$x_s^* = \frac{1}{N}\sum_{i=1}^{N} q_i y_i$$

$$y_s^* = \alpha d + \frac{(1-\alpha)a}{q_s} - \frac{(1-\alpha)}{q_s}\Big(\sum_{i=1, i\neq s}^{N}(q_i y_i - a)\Big)$$

Model: conclusions

- The work activity of a team member depends not only on his physical capabilities and propensity to consume, but also on the earnings and work intensity of other team members.
- The individual in the team works less than he would work on his own, because other members of the team with their earnings provide cost of living.

Example

- Subsistence minimum of USD 200, individual's attitude to work is 0.5, salary USD 25 / hour, limit of physical capabilities 14 hours / day, personal income tax 30%. Determine the optimal labor activity of the individual and his well-being, when he receives 100 USD and 300 USD of rental incomes.

Solution

$$\varphi(x, y) = (x - 200)^{0,5}(14 - y)^{0,5} \rightarrow \max$$

$$200 \le x \le (1 - 0,3) \cdot 25y + r,$$

$$0 \le y \le 14$$

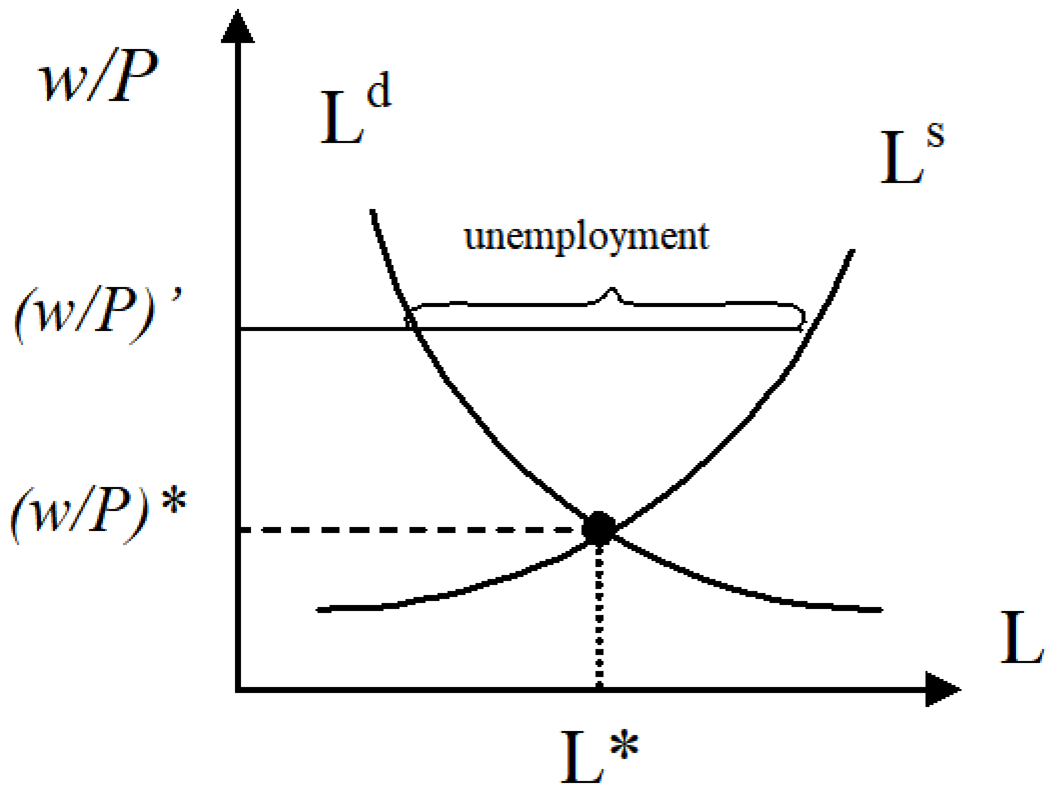
- r = 100

$$y^* = \alpha d - \frac{(1-\alpha)(r-a)}{(1-\gamma)q} = 0,5 \cdot 14 - \frac{0,5(100-200)}{0,7 \cdot 25} = 9,86$$

- r = 300

$$y^* = \alpha d - \frac{(1-\alpha)(r-a)}{(1-\gamma)q} = 0,5 \cdot 14 - \frac{0,5(300-200)}{0,7 \cdot 25} = 4,14$$

Equlibrium on the market



Causes of unemployment

- the law on the minimum wage;
- active role of trade unions in the labor market;
- theory of effective or incentive wages.

Situation in labor markets with incomplete competition – 1

- There is a firm that uses labor **monopolist in the market of finished products.**
- This leads to underutilization production resource, and the monopolist firm at the same rate hires fewer workers.
- An example: any monopoly company that operates in a large town, and therefore is not a monopolist in a labor market.

Situation in labor markets with incomplete competition – 2

- There is only one company on the market that buys factors of production – **monopsony.**
- Monopsonist has the ability to pay less at the expense of its market power in the labor market.
- An example is the only store in the village, where the administration can allow a significant reduction in the working day, as there is no competition from other stores.

Situation in labor markets with incomplete competition – 3

- It operates in the labor market **seller–monopolist.**
- The most common monopolists in the labor market are trade unions or people with unique natural abilities.
- The presence of market power allows the monopolist to impose on consumers a higher wage rate compared to a competitive market; however, this is achieved by reducing the volume of labor compared to the competitive market, ie threatens unemployment.

Situation in labor markets with incomplete competition – 4

- Market structure in which the market operates on the demand side monopsonist, and on the supply side – a monopolist, has a name **bilateral monopoly**.
- Compromise agreements are possible between the buyer and the seller in this case.

Example

- The labor market is competitive. Labor supply function:
  LS = – 2500 + 1000W,
  labor demand function
  LD = 10500 – 625W.
  Production function of the company that hires workers,
  Q = 88.8L – 0.5L2.
  Unit price = 10 UAH. Define:
  - Equilibrium wage rate and employment rate.
  - How many employees will the firm hire at the equilibrium wage rate?
  - How many products will the company produce?

Solution

$$LS = LD, -2500 + 1000W = 10500 - 625W =>$$

$$Wc = 8; Le = 5500$$

$$MP_L = \frac{\partial Q}{\partial L} = 88{,}8 - L$$

$$MRP_L = P \cdot MP_L = 10 \cdot (88{,}8 - L) = 888 - 10L$$

$$888 - 10L = 8,\ L = 88$$

$$MRP_L = W$$

$$Q = 88{,}8 \cdot 88 - 0{,}5 \cdot 88^2 = 3942$$

Theories of effective wages

- **the efforts of employees are a direct function of their wage rate.** Accordingly, the higher the wage rate, the higher productivity.
- the wage rate appears as a function of the firm's profit and as a cost of production, and as a factor that has a positive effect on revenue. Maximizing profits,**firms can choose the optimal wage rate.**
- the value of this rate **may differ from the equilibrium level**. If it exceeds the equilibrium value of wages, then the result is unemployment.

Incentive wages

There are several theories about the impact of wages on productivity:

- Food theory
- Costs casting
- Minimize labor turnover
- Adverse selection
- Sociological theory

Shapiro – Stiglitz model: description

- Based on the idea of moral hazard. Depending on the behavior of the firm, employees may have different attitudes to work, under certain conditions evading some or most of the responsibilities
- These conditions are determined by the degree of perfection of the firm's monitoring of employees, unemployment rate and wage rate
- Firms seek to set a high level of wages

Shapiro – Stiglitz model: assumption – 1

- In economics $\bar{L}$ workers and *N* firms
- Utility function of the worker

$$u(t) = w(t) - e(t),$$

where

w (t) – salary;

e (t) – the level of effort of the employee:

- e = 0, if he fades away,
- e = e> 0if he works honestly.

Model casting Shapiro – Stiglitz: assumption – 2

- The employee maximizes expectation of discounted utility

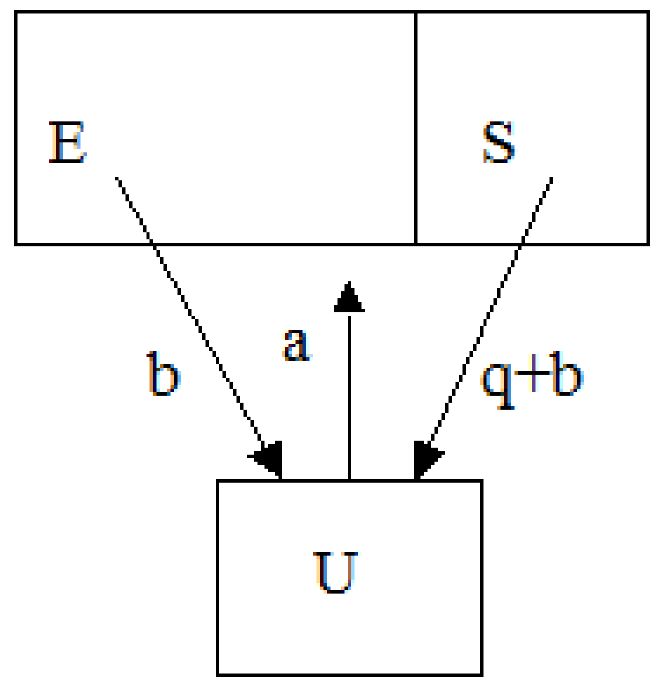
$$\int_{t=0}^{\infty} u(t)e^{-\rho t}dt$$

Where ρ is time discount.

- At any time the worker can be:
  - unemployed U,
  - “to watch” at work S,
  - work honestly E.

Shapiro – Stiglitz model : companies

- firms maximize expected profits by using efficient labor as the sole factor of production, influencing the probability of an employee moving from one state to another



Shapiro – Stiglitz model: workers

- V_i – expected discounted utility of the worker in one of 3 states

At the equilibrium point:

$$\rho V_S = w - (b + q)(V_S - V_U)$$

$$\rho V_E = (w - \overline{e}) - b(V_E - V_U)$$

$$\rho V_U = 0 + a(V_E - V_U)$$

- It will be unprofitable for the worker "to watch» ($V_E \geq V_S$) if

$$\overline{e} \leq q(V_E - V_U)$$

Shapiro – Stiglitz Model : optimum

- Wages are set at a minimum level, which encourages workers to make a high level of effort:

$$w^* = \bar{e} + (\rho + b + a) \cdot \frac{\bar{e}}{q}$$

- Demand for labor provided profit maximization

$$\bar{e}F'(\bar{e}L) = w$$

- Dynamic equilibrium condition

$$bLN = a(\bar{L} - LN)$$

- Aggregate labor supply

$$w^* = \bar{e} + (\rho + \frac{b\bar{L}}{\bar{L} - NL}) \cdot \frac{\bar{e}}{q}$$

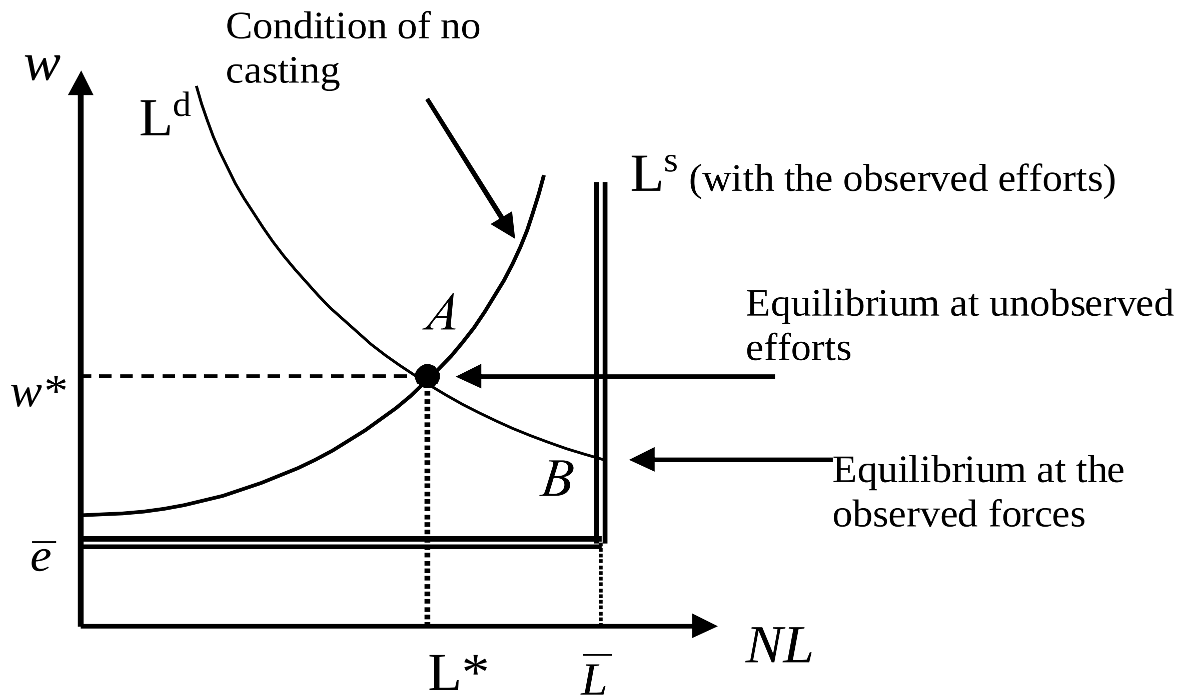
Observation by efforts

- At **observed** effort in balance employs the entire population, and the equilibrium wage exceeds the cost of effort.
- At **unobserved** efforts curve of labor supply is given by the condition of absence of “scaling”, at each point wages should be higher than the cost of effort.

Shapiro – Stiglitz model – 5

W *falls at:

- ↓ b
- ↓ and
- ↑ q
- ↑ L



Modeling the optimal contract – 1

- *w* – opportunity cost of labor at a given workplace, measured by the market wage rate equal to the marginal product of labor;
- *g* – employee benefits associated with opportunism;
- *p* – the likelihood of opportunism;
- *N* – the value of the employer-employee relationship;
- *M (p)* – the cost of control in each period.

Optimal contract modeling – 2

- The expected loss of an employee from opportunism outweighs the associated benefits

$$Np\left(w - \overline{w}\right) \geq g$$

Optimal contract modeling – 3

- Minimum wage rate at which the employee is encouraged to avoid opportunistic behavior

$$w = \overline{w} + \frac{g}{Np}$$

Optimal contract modeling – 4

- Minimize the total costs of the employer associated with preventing employee opportunism

$$M\left(p\right)+w\to\min_{p,w}$$

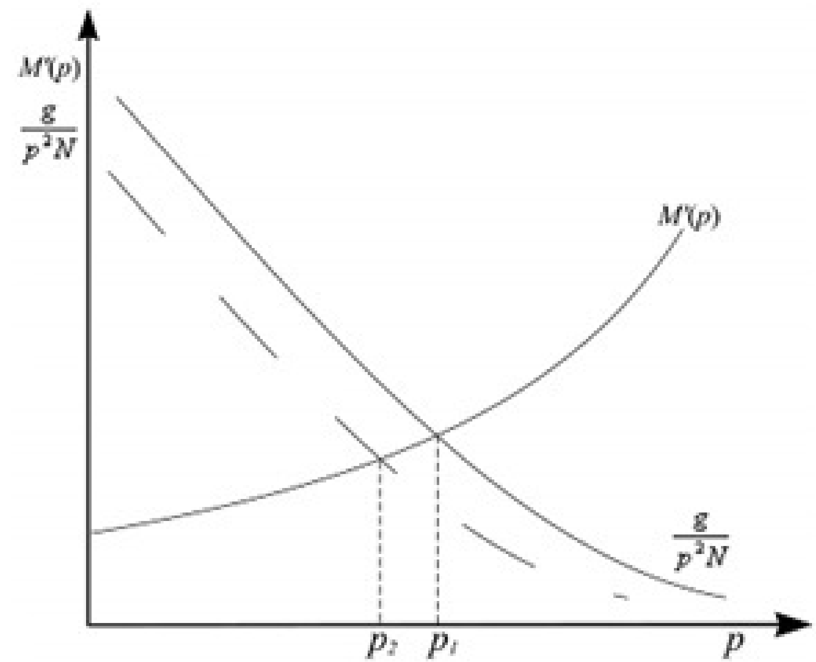
$$p\left(w-\overline{w}\right)N\geq g$$

- Marginal costs of control should be equal to the marginal benefit in the form of reduction of labor costs per 1 additional control

$$M'\left(p^*\right)=\frac{g}{p^{*2}N}$$

Optimal contract modeling – 5

- **optimal combination of control**, which minimizes the cost of control and **wages**.



Thank you!